niet uitgeven
= Voorbeeld.

DELFT UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF AERONAUTICAL ENGINEERING

Report VTH - 181

FATIGUE CRACK GROWTH UNDER VARIABLE - AMPLITUDE LOADING

by

J. Schijve

Paper presented at the Conference on the Prospects of Advanced Fracture Mechanics, Delft, The Netherlands, June 24 - 28, 1974

DELFT - THE NETHERLANDS

April 1974

DELFT UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF AERONAUTICAL ENGINEERING

Report VTH - 181

FATIGUE CRACK GROWTH UNDER VARIABLE-AMPLITUDE LOADING

by

J. Schijve

Paper presented at the Conference on the Prospects of Advanced Fracture Mechanics, Delft, The Netherlands, June 24-28, 1974

DELFT - THE NETHERLANDS

April 1974

FATIGUE CRACK GROWTH UNDER VARIABLE-AMPLITUDE LOADING

J. Schijve

Delft University of Technology, Aeronautical Department

ABSTRACT

An attempt is made to give a full description of fatigue damage, including the amount of cracking, crack front orientation, crack tip blunting, crack closure and cyclic-strain hardening in the crack tip zone. Examples of interaction effects during variable-amplitude loading are given. The usefulness and limitations of the stress intensity factor for stationary variable-amplitude loading are considered. Recent models for crack growth are surveyed. Finally the paper presents requirements and a framework for a new model, accounting for observed interaction mechanisms.

NOTATION LIST

CA	constant amplitude loading
COD	crack opening displacement
FST	flight-simulation test
K_{mF}	stress intensity for flight-simulation test, Eq. (6)
K_{rms}	root mean square of K during random load test
S_{mF}	mean stress in flight during FST
VA	variable-amplitude loading

INTRODUCTION

The propagation of fatigue cracks in an aircraft structure is an essential aspect of the fail-safe quality of the structure.

Consequently it is a design problem to ensure slow crack propagation. The main variables to achieve this goal are: (i) material selection (type of alloy, heat treatment), (ii) production techniques (e.g. integral structure or built up structure) and (iii) type of structure, dimensious (crack stopping features). After having made a choice for these variables the following problem is to predict the crack propagation rate assuming that crack nucleation has occurred. Alternative approaches are:
(a) calculations based on available data
(b) testing full-scale components
(c) compromises, such as combinations of calculations and simple tests on the material selected.

With respect to calculations considerable progress has been made for constant-amplitude loading [1, 2]. Actually the problem in that case has been reduced to calculating the stress intensity factor for a crack in a structure. The correlation between crack growth under constant-amplitude loading in a simple sheet specimen and in a complex structure by employing the stress intensity factor seems to be well established. However, in many practical cases service load histories are very much dissimilar from constant-amplitude loading, see for an example Fig. 1 [3].

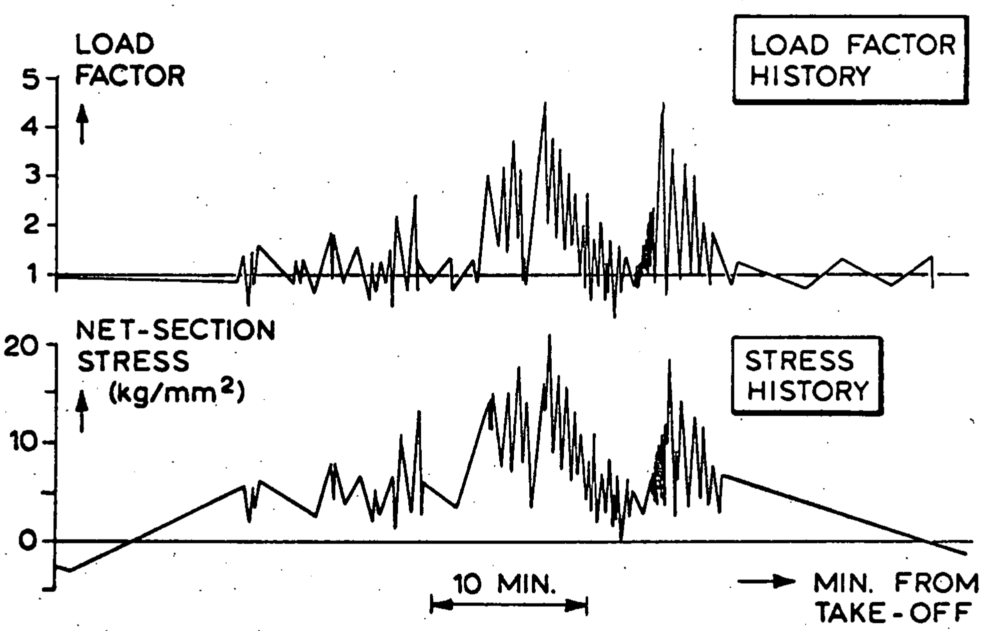


Figure 1. Service load history during a typical F-104 G flight. Maneuver loads, stress in wing. Record of minima and maxima connected by straight lines [3].

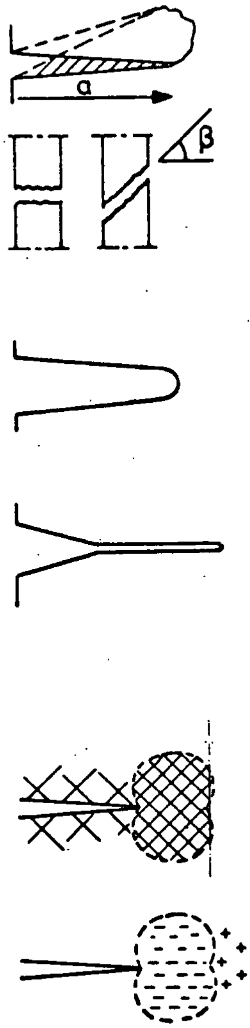
Estimation of the crack growth rate under such a loading, having available only constant-amplitude data as basic information, requires the solution of two intricate problems:
(A) The random load history has to be decomposed into a collection of discrete load cycles or ranges for which constant-amplitude data are available.
(B) A cumulative damage theory is required for predicting the crack extension in each cycle.
For both topics proposals were made in the literature [3 - 6]. Nevertheless the problem is still far from being solved.

The present paper is primarily dealing with incremental crack growth under veriable-amplitude loading, which is the background of problem B. By surveying the various aspects involved it will be shown that a theoretical crack propagation model for prediction purposes has to cope with a rather complex phenomenon. Basic requirements for a new model are indicated.

FATIGUE DAMAGE

Fatigue damage is most generally defined as the changes of the material caused by cyclic loading. Apparently the amount of cracking is a prominent aspect of these changes, but there are more changes than cracking alone. A survey is given in Fig. 2, which shows that cracking should be replaced by crack geometry, characterized by a number of complex aspects. Moreover, strain-hardening and residual stress in the crack tip zone have to be added. As a consequence a complete description of fatigue damage implies that all aspects listed in Fig. 2 should be known in quantitative terms. This must be an extremely complex description since several aspects cannot simply be indicated by a single number, consider as an example the plastic strain distribution in the crack tip zone.

The picture of fatigue damage accumulation would considerably simplify if there was a unique relation between the aspects in Fig. 2. That means if for a certain amount of cracking all other aspects b - f would always be the same, irrespective of the magnitude of the cyclic load that produced the amount of cracking. Unfortunately this situation does not apply. For example, the crack front orientation at a certain crack length will be predominantly in the tensile mode for low-amplitude loading, whereas it is in the shear mode for high-amplitude loading. Accordingly the other aspects will differ as well. It will be shown that this is important for crack growth under variable-amplitude loading.



CRACK GEOMETRY	Additional aspects
a. Amount of cracking	- crack length - shape of crack front, straight or curved
b. Crack front orientation	- tensile mode - shear mode, single shear or double shear - mixed mode
c. Crack blunting (and sharpening)	- shape of crack tip
d. Crack closure	- plastic deformation in the wake of the crack
MATERIAL CONDITION AT TIP OF CRACK	
e. (cyclic) strain-hardening	- distribution in crack tip zone
f. Residual stress	- plastic deformation in crack tip zone

Figure 2. Aspects of fatigue damage.

INTERACTION EFFECTS

In the previous section it was pointed out that fatigue damage accumulation and crack growth are not synonymous. The crack extension in a load cycle, Δa, is only one part of the damage increment in that cycle. The damage increment will be a function of: (i) the crack geometry being present before the cycle started, (ii) the condition of the crack tip material before the cycle started and (iii) the external load or stress variation to be applied in that cycle. Arguments (i) and (ii) imply that the damage increment, including Δa, will be dependent on the preceding cyclic-load history. Similarly the stress cycle applied will effect damage increments in subsequent load cycles. In the literature the dependence on the preceding history and the effect on future damage increments are referred to as "interaction effects".

Before considering the significance of interaction effects another question will be discussed first. If interaction effects would not occur, which theoretical consequences can be indicated? The absence of interaction effects implies that the damage increment cannot be dependent on all aspects listed in Fig. 2. A load cycle will simply add more damage to the damage already present. For convenience the concept "fatigue damage" may then be replaced by crack length. The size of this crack length (a) should give a sufficient indication for further damage accumulation. The damage increment in a cycle can be indicated by Δa and since it is independent of the prehistory:

$$\Delta a = da/dn \qquad (1)$$

where the crack rate da/dn is a function only of the cyclic stress applied in that particular cycle and the crack length being present. For constant-amplitude loading (CA tests, constant S_a and constant S_m), it was amply shown that the dependence can be accounted for by the stress intensity factor $\Delta K = C\ \Delta S \sqrt{\pi a} = C\ 2S_a \sqrt{\pi a}$

$$da/dn = f_R\ (\Delta K) \qquad (2)$$

The subscript R indicates that the relation is depending on the stress ratio $R = S_{min}/S_{max}$. Crack growth under variable-amplitude loading then follows from:

$$a = a_o + \Sigma\ \Delta a_i = a_o + \Sigma\ (da/dn)_i \qquad (3)$$

a_o is the initial crack length and i refers to the successive cycles of different magnitude. Equation (3) is a simple summation process of crack length increments, which are obtained as results from CA tests Eq. (2).

Some illustrations of interaction effects will now be given.
1. The crack growth delaying effect of an occasional peak load inserted into a CA test is well documented in the literature. Original explanations were based on residual stresses in the crack tip zone only. Later it became clear that crack closure [7] can also explain the delay and its contribution was clearly shown in [8].
2. In a recent test series [9] crack-opening-displacement (COD) measurements were made during flight-simulation tests. Load sequences as shown in Fig. 3 were applied. COD measurements during the larger load amplitudes are shown in Fig. 4. When the load

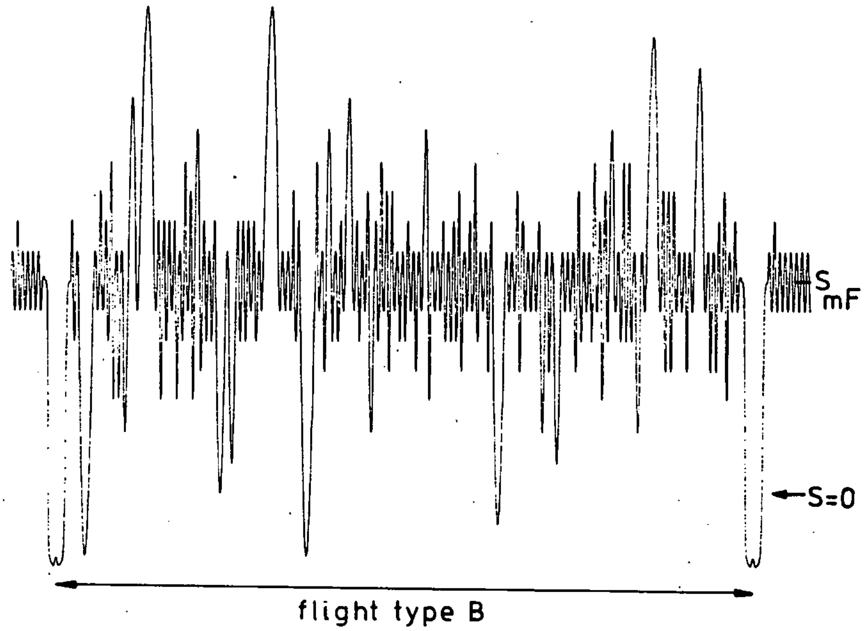


Figure 3. Load history during flight-simulation test. Flight with severe gust loads [9].

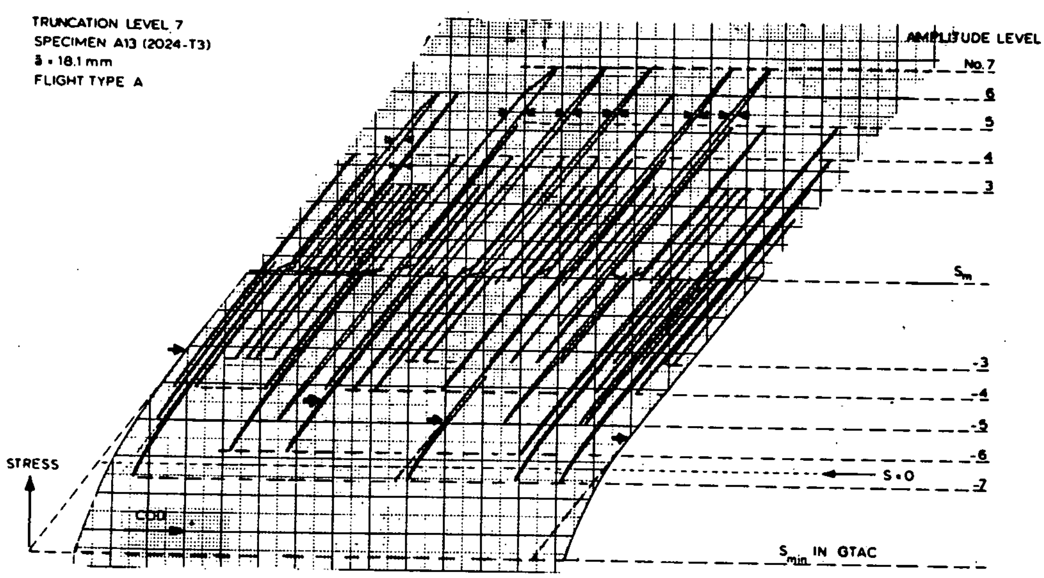


Figure 4. COD measurements during a severe flight of a flight-simulation test [9].

passed the mean stress level in flight small horizontal shifts were given to the recorder for separation of the loops of the individual loads. Two interesting observations can be made. First, the five positive peak loads occurring in the same flight do not have the same effect. The non-linearity at the top of the former one is mainly due to crack extension as shown by crack growth observations. Apparently the other peak loads give a much smaller contribution. This is in agreement with results from a more elementary study [10], where it was shown that Δa during the first peak load was much larger than da/dn corresponding to the size of load cycles (accelerated crack growth). It was argued that incompatible crack front orientation and crack sharpening and strain-hardening during the preceding cycles (aspects b, c and e in Fig. 2) may have contributed to this phenomenon.
The second observation is related to crack closure which is clearly evident from the non-linearity in the lower part of the recording. Arrows indicate the stress levels below which the crack is partly closed. The first high load, causing crack extension and plastic elongation of the crack tip zone, has decreased the crack closure stress. After further crack growth during subsequent flights the crack closure stress was again restored to a higher level. This once again confirms the occurrence of crack closure.
3. In Fig. 5 an example of incompatible crack front orientation is shown.

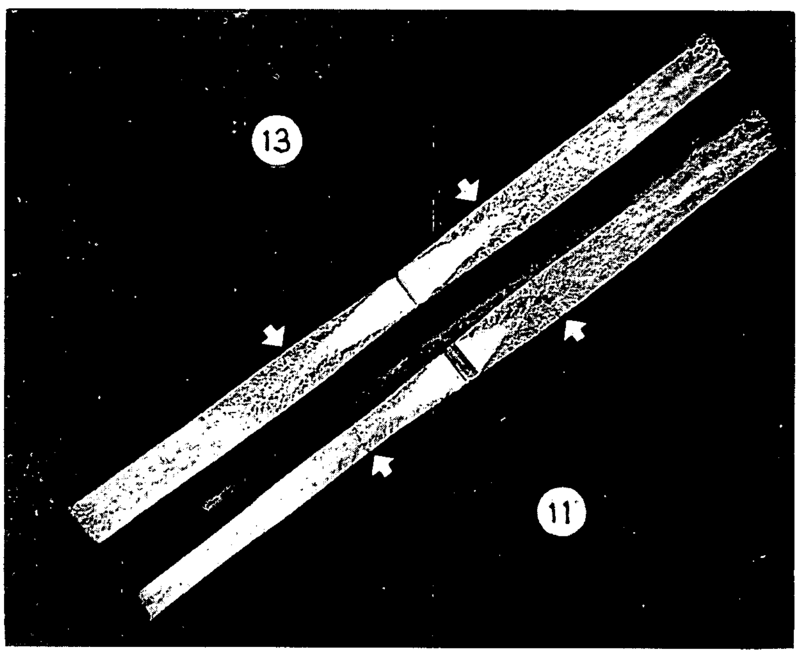


Figure 5. Fracture surfaces (specimen thickness 4 mm) showing incompatible crack front orientation (arrows) during an intermittent block of low-amplitude cycles [10].

The high-amplitude loading caused the transition from the tensile mode to the single shear mode. When the latter one was present a block of low-amplitude cycles was applied. In a CA tests with the latter cycles only the crack front at the crack lengths indicated by arrows was still in the tensile mode. In other words there was a mismatch between the low amplitude loading and crack front orientation. The crack rate during the low-amplitude cycles was far below the corresponding value of the CA tests (delayed crack growth). In addition to the incompatibility of the crack front orientation other aspects may have contributed also (aspects c - f in Fig. 2).

4. Sequence effects have been observed in variable-amplitude test series. In a recent investigation [11] the same variable-amplitude cycles were applied in a random sequences and in a programmed sequence. Crack growth was about three times faster for the former case.

PREDICTION PURPOSES

Some remarks should be made on the goals to be achieved and the accuracies required. Two classes of problems can be defined:

1. Prediction of trends. In these types of problem the effect of a certain variable is looked for. Some examples will be mentioned.

1a. The load spectrum in service may differ from the load spectrum applied in a full-scale fatigue test, during which crack propagation data were obtained. For instance, it may contain relatively less high-amplitude cycles, or relatively more low-amplitude cycles. The question is, how should the full-scale test data be corrected for these effects.

1b. Comparing two alternative designs for the same structure, for instance an old design and a redesign. The question is how much slower the crack propagation will be in the redesigned structure.

1c. Considering two alloys for material selection a relevant question is how the propagation rates under the service load history will compare.

In these practival problems a comparison has to be made between two or more different cases. It would be very helpful already if the order of magnitude of the effect of the variable can be indicated. This implies that high accuracies appears to be unnecessary.

2. Estimation of the crack propagation rate. For a specific structure the crack propagation rate for the anticipated service load spectrum has to be estimated. This has to be known in view of fail-safe consideration. In this case a better accuracy should be achieved than for predicting the effect of trends.

INTERACTION MODELS FROM THE LITERATURE

Non-interaction models are incorrect from a physical point of view, but one might hope that predictions with Eq. (3) could still give useful indications for problems as listed before. Unfortunately the non-interaction concept can highly misjudge the effect of certain variables and even the sign of the predicted trend may be incorrect [9, 12]. The simple conclusion is that there is an apparent need for a more reliable prediction method.

Interaction models for crack propagation were proposed by Willenborg, Engle and Wood [4], by Habibie [5] and by Wheeler [6]. Although these models are different they have some aspects in common:

1. Crack growth increments (Δa) are calculated cycle by cycle.
2. The crack growth increment in each cycle is correlated to the increment that would have been obtained in a constant-amplitude test (CA data) at the same crack length.
3. Mathematically Habibie and Wheeler employ a delay factor φ defined by

$$\left(\frac{da}{dn}\right)_{VA,\ a=a_i} = \varphi \cdot \left(\frac{da}{dn}\right)_{CA,\ a=a_i} \quad (4)$$

(VA = variable amplitude test.) Willenborg et al employ a reduced $K_{max.}$ value. All three models basically consider delay effects due to high peak loads only.

The essential elements of the models were recently compared by Broek [13]. He indicated the similarity of the models of Habibie and Wheeler, expressing some preference for the Wheeler model in view of its more clear physical conception. He also mentioned the doubtful assumption about the reduced K value in the Willenborg model.

Habibie developped his method especially for flight load conditions. He introduces several empirical constants (α, γ, ξ_i, η_i) for calculating the delay factor φ. These constants were derived from comparisons between flight-simulation test data and non-interaction predictions. In view of the many empirical constants the method should be expected to be more flexible in predicting results for similar loading conditions. However, for other conditions its usefulness have to be proven again, because the method is largely an empirical method.

In the Wheeler model and the Willenborg model the dimensions of the plastic zones are calculated with K-values, assuming that the same yield stress applies in any case. The Willenborg model does not employ any additional constants, whereas the Wheeler model includes one more constant m. The latter constant gives some flexibility for matching data, but unfortunately the m-value required for that purpose may be somewhat unsystematic [14].

The three models consider delay effects only, which are assumed to be due to crack tip plasticity. This involves severe limitations, examples being listed below.

1. Crack closure does not occur in the above models. This implies that the observations of Van Euw et al [8] on the effect of overloads cannot be explained. The significance of this observation for the comparison between programmed load sequences and random sequences was recently stressed by Katcher [14].

2. Incompatible crack front orientation as discussed before does not occur in the above models. However, it was shown to be significant [10].

3. Crack tip blunting and sharpening should have some effect on the distribution of stress and strain in the crack tip zone, but they do not occur in any model.

4. Cyclic strain-hardening is another aspect ignored by the above models. According to these models multiple overloads, applied in one block, should have a harmfull effect as compared to a single overload. The delay would be the same, but the crack growth increment of the overloads would be larger. However, test results of Hudson and Raju [15] indicated that a batch of overloads gives a longer crack growth delay than a single overload. This may well be attributed to an increased cyclic hardening, although the full explanation may be more complex.

A physically sound model should be able to deal with all above observations and not with just one of them. This requirement leads to a complex formalization of a model as will be shown later on.

THE STRESS INTENSITY FACTOR APPLIED TO VARIABLE-AMPLITUDE LOADING

The application of K for correlating data from constant-amplitude tests has been very succesful so far. Unfortunately crack growth delays cannot be reconciled with Eq. (2), unless further refinements are introduced [16]. However, it is difficult to see how simple adaptions to the K-concept could explain the observations discussed before. It is then a promising result that crack propagation under stationary random loading can be correlated if S_{rms} is substituted into K [17-19] , as proposed by Paris [20].

$$K_{rms} = C\, S_{rms} \sqrt{\pi a} \tag{5}$$

The basic idea is that the random variations of the stresses in the crack tip zone are fully described by K_{rms}.

Recently [21] an attempt has been made to apply the same idea to flight-simulation tests (Fig. 3). Actually the loading is a mixture of random gust loads and deterministic ground-to-air cycles. Nevertheless the loading has a statistically stationary character. New load histories were obtained by amplifying all stress levels with the same factor, without changing the load sequence. In engineering terms this means that the design stress level was changed. For that reason one stress value fully describes the intensity of the load history in these test series. Taking the mean stress in flight S_{mF} (Fig. 3), as such a value it was hoped that K_{mF} would correlate the crack propagation data.

$$K_{mF} = C\ S_{mF}\sqrt{\pi a} \qquad (6)$$

Results for the 7075-T6 alloy are shown in Fig. 6. A similar graph was obtained for the 2024-T3 alloy. Unfortunately the results indicate that a unique correlation

$$(da/dn)_{FST} = f\ (K_{mF}) \qquad (7)$$

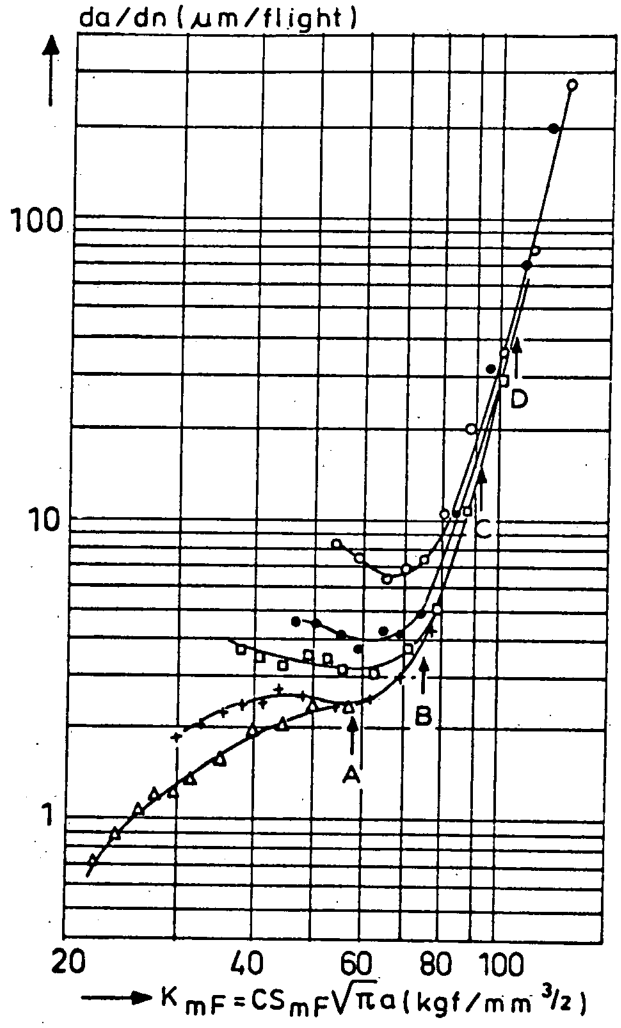


Figure 6. Crack propagation rates (microns per flight) as obtained in flight-simulation tests [21].

apparently does no longer hold true. This is indeed most unfortunate because the validity of Eq. (7) would have allowed a simple extrapolation from one stress level to another level. Several practical applications could have been foreseen.

Since ΔK and K_{rms} are applicable to crack growth under CA and random loading, it remains to be explained why K_{mF} fails to correlate the data in Fig. 6. The basic idea of the application of K to fatigue crack growth is: two specimens with different crack length and different cyclic loads, but the same instanteneous K-values, should have the same crack growth rate [20]. This similarity, however, will only be valid if the material of the two specimens is the same. In more detail, the material in the plastic zone at the tip of the crack should be the same. This requires that cyclic-strain hardening and residual stresses at the tip of the crack should be the same. However, they are depending on the previous cyclic-strain history. The significance of this aspect will be illustrated by discussing Fig. 7.

Figure 7 shows the relation between K and a for two specimens, fatigue tested at a high and a low stress level respectively. At a crack length a_1 in specimen 1 and a crack length a_2 in specimen 2 the same K-value applies and one might expect similar crack rates. At $a_1 + \Delta a_1$ and $a_2 + \Delta a_2$ similar crack rates should again be expected. However, since $\Delta a_2 > \Delta a_1$ the cyclic-strain histories at the tips of the two cracks cannot have been the same. A similar cyclic-strain history would require the two curves to be parallel.

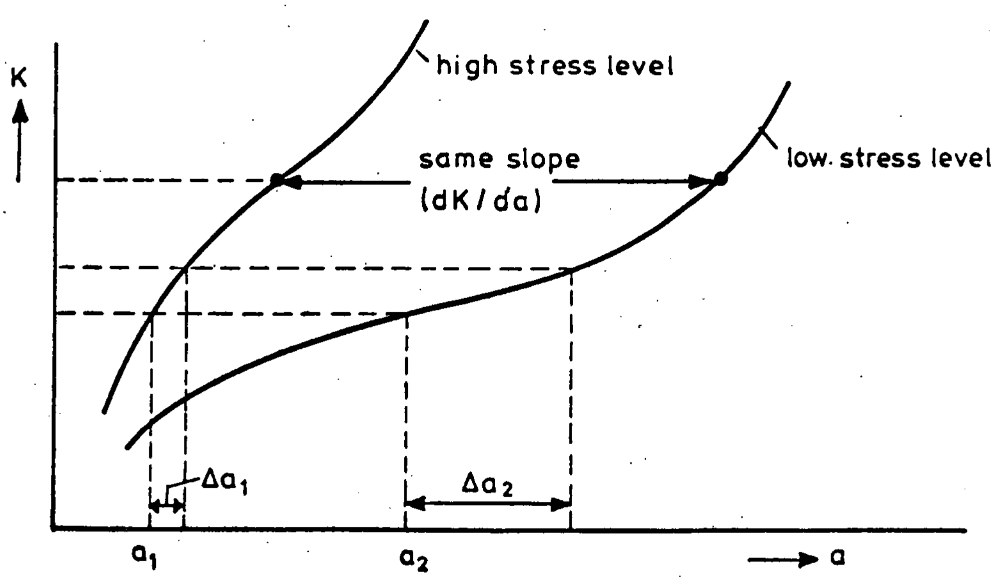


Figure 7. K as a function of crack length for two different stress levels [21].

More specifically, similar crack rates in two specimens may be expected if both K and dK/da are similar. Unfortunately these two requirements are generally incompatible, and hence Eq. (2) should be rewritten as:

$$da/dn = f_R\ (\Delta K,\ dK/da) \quad (8)$$

As illustrated by Fig. 7 equal dK/da values will occasionally occur for the same K-value and two different stress levels. Such K-values, calculated in [21], are indicated in Fig. 6 by arrows. It is noteworthy that equal da/dn values are apparently obtained then. This gives some support to Eq. (8).

It follows from the previous reasoning that the validity of $da/dn = f_R\ (\Delta K)$ for CA tests, independent of dK/da, should not be appreciated as an obvious law, but rather as a kindness of nature. In [20] Paris pointed out that the load history at a certain point in the material through which the crack will pass may be important. He emphasized that a relation between da/dn and K for any type of loading should be confirmed by tests. If da/dn is uniquely correlated to K this implies that the effect of dK/da in Eq. (8) is negligible. Apparently this applies to CA loading, although dK/da values may be highly different. An encouraging result [22] was obtained in a comparison between specimens with end-loading and specimens with wedge-force loading. The value of dK/da is positive for the former loading and negative for the latter one. Nevertheless da/dn could well be correlated by K alone. The cyclic strain-history in the plastic zone at the tip of the crack is changing only slowly from cycle to cycle in these tests. That may explain why differences in the cyclic-strain history (due to different dK/da values) do not affect the crack rate.

Another encouraging result was the applicability of K_{rms} in Eq. (5). In [21] it was pointed out that random loading with a Rayleigh distribution for the higher peak loads could be relatively free from interactions effects. In flight-simulation tests, however, such effects are known to be present [12]. Under such conditions the load history in the plastic zone becomes important and an effect of dK/da should be expected.

REQUIREMENTS FOR A NEW MODEL

The previous discussion indicates that significant interaction effects do occur during variable-amplitude loading. These effects are caused by a number of different mechanisms. Basically a reliable model should incorporate all these mechanisms. An attempt will now be made to indicate the analytical consequences of this requirement.

Crack extension occurs during a load increase only. The amount of crack extension (Δa) is supposed to be a function of the plastic deformation in the crack tip zone (ε^+).

$$\Delta a = f_1\ (\varepsilon^+) \tag{9}$$

The plastic deformation in the crack tip zone will be a function of the local stress variation (ΔS_{tip}) and the local yield stress ($S_{yield,\ tip}$), which is depending on the local strain hardening.

$$\varepsilon^+ = f_2\ (\Delta S_{tip}, S_{yield,\ tip}) \tag{10}$$

As a first approximation crack closure will be accounted for by assuming that the crack tip zone remains unaffected by the external loading until the stress has exceeded the stress level to fully open the crack ($S_{c.o.}$). It will also be affected by the crack tip geometry (r_{tip}) and the crack front orientation (angle β).

$$\Delta\ S_{tip} = f_3\ (S_{max} - S_{c.o.}, r_{tip}, \beta) \tag{11}$$

In order to calculate Δa from Eq. (9) by substituting Eqs. (10) - (11) the instantaneous values of $S_{yield,\ tip}$, $S_{c.o.}$, r_{tip} and β have to be known. They all depend on the preceding history. $S_{yield,\ tip}$ and r_{tip} will change from cycle to cycle, while both are dependent on the local strain history, indicating that the two quantities are interdependent.

$$S_{yield,\ tip} = f_4\ (\text{local strain history}) \tag{12}$$

$$r_{tip} = f_5\ (\text{local strain history}) \tag{13}$$

On the other hand the crack opening stress is a function of the plastic strain history in the wake of the crack. The same will apply to β.

$$S_{c.o.} = f_6\ (\text{strain history along the crack}) \tag{14}$$

$$\beta = f_7\ (\text{strain history along the crack}) \tag{15}$$

The local strain history is an accumulation of positive and negative strain increments. Indicating the local position by x=a and the cycle number by i the local strain history is characterized by the sum of the strain increments during load increases (ε^+) and decreases (ε^-).

$$\text{local strain history} \mathrel{\hat{=}} \sum_{i=1}^{n} \varepsilon^+_{x,i}, \varepsilon^-_{x,i}\ (x=a) \tag{16}$$

Making the summation for all positions along the crack one obtains the strain history along the crack.

$$\text{strain history along crack} \mathrel{\hat{=}} \sum_{i=1}^{n} \varepsilon^+_{x,i}, \varepsilon^-_{x,i}\ (x) \tag{17}$$

In order to calculate strain histories an equation for ε^- similar to Eq. (9) is still required. Assuming that all above equations were available the crack extension in a load cycle (Δa) can be calculated. First the strain histories of the preceding load cycles are calculated, Eqs. (16) and (17). Substitution in Eqs. (12) - (15) gives the input data for Eqs. (10) and (11) and Δa then follows from Eq. (9).

Obviously the model outlined above requires highly complex calculations depending on the functional relations. In the above equations ε^+, ε^- and $S_{yield, tip}$ were tacitly supposed to be single-valued variables. However, in reality they should be characterized by a spatial distribution. Consequently, to arrive at a feasible model some simplifications will be necessary. It then should be kept in mind whether the simplification appears to be acceptable in view of interaction mechanisms, which can be active. A first simplification may be the omission of β, because the model could still be relevant to crack propagation occuring predominantly in plane strain (β=0). More simplifications may be considered, however, the models presented in the literature were simplified too rigorously. As a consequence crack growth accelerations became impossible.

CONCLUDING REMARKS

1. During variable-amplitude fatigue several interaction mechanisms can exert a significant effect on crack growth. As a result non-interaction techniques are incorrect for the prediction of crack propagation rates. These techniques are even unreliable for indicating trends when comparing different conditions. Recently published crack growth models have been based on crack growth delays caused by high peak loads. Unfortunately, the models do not pay much attention to the physical mechanism of this interaction. Actually the starting point is too restrictive for having confidence in a broader application beyond the conditions of the tests, which served to determine the empirical constant(s) of the model.
2. A reliable model for crack growth should account for all interaction mechanisms that are known to exert significant effects. The aim of the model outlined in this paper is a preliminary framework for this purpose. An attempt was made to incorporate the present knowledge about fatigue damage accumulation, but it cannot be denied that the problem is still open to discussion. The derivation of the various function is still another tremendous problem. Nevertheless, it is thought that a more rational model is a challenging possibility by now.
3. The stress intensity factor ΔK produced excellent correlations for crack growth under constant-amplitude loading. For stationary random loading the same applies to K_{rms}, although the empirical evidence is still somewhat limited. Unfortunately for stationary flight-simulation loading K_{mF} was unsuccessful in correlating the

crack growth data. This has led to a re-evaluation of the K-concept indicating that dK/da may also effect crack growth for stationary load histories, especially if significant interaction effects occur.

REFERENCES

[1] Poe, C.C., Fatigue crack propagation in stiffened panels. ASTM STP 486,79, 1971.
[2] Salvetti, A. and Del Puglia, A., Fatigue crack propagation in stiffened panels. ICAS Paper 72-40, 1972.
[3] Van Dijk, G.M., Statistical load data processing. NASA SP-309, 1972, 565.
[4] Willenborg, J.D., Engle, R.M. and Wood, H.A., A crack growth retardation model using an effective stress concept. AFFDL-TM-FBR-71-1, 1971.
[5] Habibie, B.J., Eine Berechnungsmethode zur Voraussage des Fortschritts von Rissen unter beliebigen Belastungen und Vergleiche mit Entsprechenden Versuchsergebnissen. Paper DGLR meeting, Baden-Baden, 1971.
[6] Wheeler, O.E., Spectrum loading and crack growth. J. Basic Engng, Trans. ASME, 181, 1972.
[7] Elber, W., The significance of fatigue crack closure. ASTM STP 486, 230, 1971.
[8] Von Euw, E.F.J., Hertzberg, R.W. and Roberts, R., Delay effects in fatigue crack propagation. ASTM STP, 513, 230, 1972.
[9] Schijve, J., Jacobs, F.A. and Tromp, P.J., Flight-simulation tests on notched elements. NLR TR 74033, 1974.
[10] Schijve, J., Fatigue damage accumulation and incompatible crack front orientation. Engng. Fracture Mech. (to be published shortly).
[11] Schijve, J., Effect of load sequences on crack propagation under random and program loading. Engng. Fracture Mech., 5, 269, 1973.
[12] Schijve, J., The accumulation of fatigue damage in aircraft materials and structures. AGARDograph No. 157, 1972.
[13] Broek, D., Faile-safe design procedures, NLR MP 72021, 1972.
[14] Katcher, M., Crack growth retardation under aircraft spectrum loads. Engng. Fracture Mech., 5, 793, 1973.
[15] Hudson, C.M. and Raju, K.N., Investigation of fatigue-crack growth under simple variable-amplitude loading. NASA TN D-5702, 1970.
[16] Crews, J.H. and White, N.H., Fatigue crack growth from a circular hole with and without high prior loading. NASA TN D-6899, 1972.
[17] Paris, P.C., The fracture mechanics approach to fatigue, in Fatigue, an Interdiciplinary Approach, Syracuse Un. Press, 1964, 107.
[18] Smith, S.H., Fatigue crack growth under axial narrow and broad band random loading, in Aconstical Fatigue in Aerospace Structures, Syracuse Un. Press, 1965, 331.

[19] Swanson, S.R., Cicci, F. and Hoppe W., Crack propagation in Clad 7079-T6 aluminium alloy sheet under constant and random amplitude fatigue loading. ASTM STP 415, 312, 1967.
[20] Paris, P.C., The growth of cracks due to variations in load. Doctor thesis, Lehigh Un., 1962.
[21] Schijve, J., The prediction of fatigue crack propagation under service load-time histories. Paper ICAF Symp. London, July '73.
[22] Figge, I.E. and Newman, J.C., Fatigue crack propagation in structures with simulated rivet forces. ASTM STP 415, 71, '67.